EXERCISE – V | JEE PROBLEMS

1. Select the correct alternative
[JEE 2000(Scr.), 1 + 1 + 1]

(i) If the vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$ form the sides BC, CA & AB respectively of a triangle ABC, then
(A) $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = 0$
(B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
(C) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} \cdot \vec{a}$
(D) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$

(ii) Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ be such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. Let P_1 & P_2 be planes determined by the pairs of vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$, $\vec{d}$ respectively. Then the angle between P_1 and P_2 is
(A) 0 (B) $\pi/4$ (C) $\pi/3$ (D) $\pi/2$

(iii) If $\vec{a}$, $\vec{b}$ & $\vec{c}$ are unit coplanar vectors, then the scalar triple product $[2\vec{a} - \vec{b}\ \ 2\vec{b} - \vec{c}\ \ 2\vec{c} - \vec{a}] =$
(A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$

2. (i) If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ & $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$, find a unit vector normal to the vectors $\vec{a}+\vec{b}$ and $\vec{b}-\vec{c}$. **[REE 2000(Mains), 3 + 3 + 3]**

(ii) Given that vectors $\vec{a}$ & $\vec{b}$ are perpendicular to each other, find vector $\vec{v}$ in terms of $\vec{a}$ & $\vec{b}$ satisfying the equations, $\vec{v}\cdot\vec{a} = 0$, $\vec{\upsilon}\cdot\vec{b} = 1$ and $[\vec{v}\ \vec{a}\ \vec{b}] = 1$

(iii) $\vec{a}$, $\vec{b}$ & $\vec{c}$ are three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}(\vec{b} + \vec{c})$. Find angle between vectors $\vec{a}$ & $\vec{b}$ given that vectors $\vec{b}$ & $\vec{c}$ are non-parallel.

3. (a) The diagonals of a parallelogram are given by vectors $2\hat{i} + 3\hat{j} - 6\hat{k}$ and $3\hat{i} - 4\hat{j} - \hat{k}$. Determine its sides and also the area. **[REE 2001(Mains), 3 + 3]**

(b) Find the value of λ such that a, b, c are all non-zero and
$(-4\hat{i}+5\hat{j})a + (3\hat{i}-3\hat{j}+\hat{k})b + (\hat{i}+\hat{j}+3\hat{k})c = \lambda(a\hat{i}+b\hat{j}+c\hat{k})$

4. (a) Find the vector $\vec{r}$ which is perpendicular to $\vec{a} = \hat{i} - 2\hat{j} + 5\hat{k}$ & $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ & $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$
[REE 2001(Mains), 3 + 3]

(b) Two vertices of a triangle are at $-\hat{i}+3\hat{j}$ and $2\hat{i}+5\hat{j}$ and its orthocentre is at $\hat{i}+2\hat{j}$. Find the position vector of third vertex.

5. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$ does NOT exceed
[JEE 2001(Scr.), 1 + 1]
(A) 4 (B) 9 (C) 8 (D) 6

(b) Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on
(A) only x (B) only y
(C) neither x nor y (D) both x and y

6. Let $\vec{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$ and $\vec{B}(t) = g_1(t)\hat{i} + g_2(t)\hat{j}$, $t \in [0, 1]$, where f_1, f_2, g_1, g_2 are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are nonzero vectors for all t and $\vec{A}(0) = 2\hat{i} + 2\hat{j}$, $\vec{A}(1) = 6\hat{i} + 2\hat{j}$, $\vec{B}(0) = 3\hat{i} + 2\hat{j}$ and $\vec{B}(1) = 2\hat{i} + 6\hat{j}$, then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t. **[JEE 2001(Mains), 5]**

7. (a) If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is **[JEE 2002(Scr.), 3 + 3]**
(A) 45º (B) 60º
(C) $\cos^{-1}(1/3)$ (D) $\cos^{-1}(2/7)$

(b) Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{W} = \hat{i} + 3\hat{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is
(A) –1 (B) $\sqrt{10} + \sqrt{6}$ (C) $\sqrt{59}$ (D) $\sqrt{60}$

8. If $\vec{a} = \hat{i} + a\hat{j} + \hat{k}$, $\vec{b} = \hat{j} + a\hat{k}$, $\vec{c} = a\hat{i} + \hat{k}$, then find the value of 'a' for which volume of parallelopiped formed by three vectors as coterminous edges, is minimum, is **[JEE 2003(Scr.), 3]**

(A) $\frac{1}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\pm\frac{1}{\sqrt{3}}$ (D) none

9. If $\vec{u}$, $\vec{v}$, $\vec{w}$ are three non-coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}$, $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$ respectively and $\vec{x}$, $\vec{y}$, $\vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. Prove that $[\vec{x} \times \vec{y} \;\; \vec{y} \times \vec{z} \;\; \vec{z} \times \vec{x}] = \frac{1}{16}[\vec{u} \;\; \vec{v} \;\; \vec{w}]\sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$. **[JEE 2003, 4]**

10.(a) A unit vector in the plane of the vectors $2\hat{i} + \hat{j} + \hat{k}, \hat{i} - \hat{j} + \hat{k}$ and orthogonal to $5\hat{i} + 2\hat{j} + 6\hat{k}$ **[JEE 2004(Scr.)]**

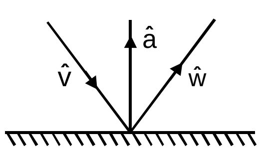
(A) $\frac{6\hat{i} - 5\hat{k}}{\sqrt{61}}$ (B) $\frac{3\hat{j} - \hat{k}}{\sqrt{10}}$ (C) $\frac{2\hat{i} - 5\hat{k}}{\sqrt{29}}$ (D) $\frac{2\hat{i} + \hat{j} - 2\hat{k}}{3}$

(b) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b}$ equals

(A) $\hat{i}$ (B) $\hat{i} - \hat{j} + \hat{k}$ (C) $2\hat{j} - \hat{k}$ (D) $2\hat{i}$

11. Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are four distinct vectors satisfying $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$. Show that $\vec{a}.\vec{b} + \vec{c}.\vec{d} \neq \vec{a}.\vec{c} + \vec{b}.\vec{d}$. **[JEE 2004, 2]**

12. Incident ray is along the unit vector $\hat{v}$ and the reflected ray is along the unit vector $\hat{w}$. The normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$.
[JEE 2005 (Mains), 4]



13.(a) Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ has the magnitude equal to $1/\sqrt{3}$, is **[JEE 2006, 3 + 5]**

(A) $4\hat{i} - \hat{j} + 4\hat{k}$ (B) $3\hat{i} + \hat{j} - 3\hat{k}$

(C) $2\hat{i} + \hat{j} - 2\hat{k}$ (D) $4\hat{i} + \hat{j} - 4\hat{k}$

(b) Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

14. (a) The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is **[JEE 2007, 3+3+3]**

(A) zero (B) one (C) two (D) three

(b) Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Which one of the following is correct ?

(A) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{0}$

(B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$

(C) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{a} \times \vec{c} \neq \vec{0}$

(D) $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ are mutually perpendicular.

(c) Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Statement-I : $\overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq \vec{0}$

because

Statement-II : $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$

(A) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I
(B) Statement-I is true, statement-II is true; statement-II is **NOT** a correct explanation for statement-I
(C) Statement-I is true, Statement-II is False
(D) Statement-I is False, Statement-II is True

15. (a) The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}$, $\hat{b}$, $\hat{c}$ such that $\hat{a} \cdot \hat{b} = \hat{b} \cdot \hat{c} = \hat{c} \cdot \hat{a} = \frac{1}{2}$. Then the volume of the parallelopiped is **[JEE 2008, 3+3]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

(b) Let two non-collinear unit vector $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}$ cos t + $\hat{b}$ sin t. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then,

(A) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and M = $(1 + \hat{a}.\hat{b})^{1/2}$

(B) $\hat{u} = \dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and M = $(1 + \hat{a}.\hat{b})^{2}$

(C) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and M = $(1 + 2\hat{a}.\hat{b})^{1/2}$

(D) $\hat{u} = \dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and M = $(1 + 2\hat{a}.\hat{b})^{1/2}$

16.(a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are unit vectors such that $((\vec{a}\times\vec{b}).(\vec{c}\times\vec{d}) = 1$ and $\vec{a}.\vec{c} = \dfrac{1}{2}$, then **[JEE 2009, 3+3+3+8+4]**

(A) $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar

(B) $\vec{b}$, $\vec{c}$, $\vec{d}$ are non-coplanar

(C) $\vec{b}$, $\vec{d}$ are non-parallel

(D) $\vec{a}$, $\vec{d}$ are parallel and $\vec{b}$, $\vec{c}$ are parallel

(b) Match the statements/expressions given in **Column–I** with the value given in **Column–II.**

Column–I	**Column–II**
(A) Roots(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(P) $\dfrac{\pi}{6}$
(B) Points of discontinuity of the function $f(x) = \left[\dfrac{6x}{\pi}\right]\cos\left[\dfrac{3x}{\pi}\right]$, where [y] denotes the largest integer less than or equal to y	(Q) $\dfrac{\pi}{4}$
(C) Volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(R) $\dfrac{\pi}{3}$
(D) Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\,\vec{c}=\vec{0}$	(S) $\dfrac{\pi}{2}$
	(T) π

17. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ & $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a **[JEE 2010]**

(A) parallelogram, which is neither a rhombus nor a rectangle

(B) square

(C) rectangle, but not a square

(D) rhombus, but not a square

18. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \dfrac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \dfrac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a}+\vec{b}).[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})]$ is **[JEE 2010]**

19. Two adjacent sides of a parallelogram ABCD are given by $AB = 2\hat{i}+10\hat{j}+11\hat{k}$ and $AD = -\hat{i}+2\hat{j}+2\hat{k}$

The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by **[JEE 2010]**

(A) $\dfrac{8}{9}$ (B) $\dfrac{\sqrt{17}}{9}$ (C) $\dfrac{1}{9}$ (D) $\dfrac{4\sqrt{5}}{9}$

20. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = \hat{i}-\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\dfrac{1}{\sqrt{3}}$, is given by **[JEE 2011]**

(A) $\hat{i}-3\hat{j}+3\hat{k}$ (B) $-3\hat{i}-3\hat{j}-\hat{k}$

(C) $3\hat{i}-\hat{j}+3\hat{k}$ (D) $\hat{i}+3\hat{j}-3\hat{k}$

21. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are **[JEE 2011]**

(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

22. Let $\vec{a} = -\hat{i}-\hat{k}, \vec{b} = -\hat{i}+\hat{j}$ and $\vec{c} = \hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is **[JEE 2011]**

23. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a} + \vec{b}| = \sqrt{29}$ and $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$ then a possible value of $(\vec{a} + \vec{b}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$ is **[JEE 2012]**

(A) 0 (B) 3 (C) 4 (D) 8

24. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9$, then $\left|2\vec{a} + 5\vec{b} + 5\vec{c}\right|$ is

[JEE 2012]